Crowd Health Encoding, for Crowd Simulations Using the Smoothed Particle Hydrodynamics Computational Method

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Abstract. Crowd injury modelling focuses on investigating dangerous scenarios involving crowds, including incapacitating, altering behaviour of, or otherwise disturbing individuals in a way which affects the simulation's motion and status. In this paper, multiple simulated crowd properties are combined into a single model, and a subsequent prototype is designed and produced using recent methods of crowd simulation, to investigate the usage of persistent, proactive, simulated crowd monitoring of dangerous scenarios, for providing crowd management studies with additional safety information about potentially hazardous pedestrian crowd gatherings. "Crowd health" is an introduced term for the involvement of properties that affect a person's, or their neighbours', health within a crowd. Properties considered are panic, which may affect movement, and crush, which may cause injuries. Results of experimentation inform the selection and variation of parameters to produce characteristics of, and observe changes in, crowd health, while maintaining simulation stability and control using the Smoothed Particle Hydrodynamics technique. Findings include defining a way to modify a recent crowd simulation model to inform safety by reporting dynamic crowd health.

Keywords: Crowd Simulation · Smoothed Particle Hydrodynamics · Particle Simulation · Injury Simulation · Crowd Health.

1 Introduction

Crowd simulation was developed from agent-based simulations, to be used mainly for robotics and entertainment applications. Later, crowd injury modelling focused on investigating dangerous scenarios concerning crowds. Individual scenarios are analysed by imitating past hazardous crowd occurrences, considering external hazards to the crowd like in evacuations, or using post-simulation crowd flow data to represent crowd density, limiting the information gained useful to prevent such scenarios in the future. This demonstrates a need to adapt real-time crowd simulators for providing verifiable, safety-critical, predictive information about internal crowd behaviour useful before the hazard really occurs.

To simulate crowd motion trustable enough to inform safety means shifting the model focus from providing stability and believability, to thorough health-based insight into the real world. For example, some simulated properties may be inferred as instability or erratic human behaviour. This distinction reveals a research goal between artificial crowd simulation and real-world crowd analysis.

In this study, as explained in the methodology section, a simulation model is described in the design section, using insight gained from the background research section. Then, a subsequent prototype is created and validated within the experimentation section. Using this prototype we will investigate the use of metrics involved in crowd simulation, and a recently developed Smoothed Particle Hydrodynamics (SPH) model, for representing the presence, development, and alteration of aspects of health within a simulated crowd. SPH is a technique relatively unused for crowd motion, but suitably applied to dense crowds, which still has applications to investigate within the subtopics of crowd simulation; it's presence could help represent internal forms of dense-crowd health.

To encapsulate the nature of this new model, the term "crowd health simulation" is introduced to combine the behaviours and dynamic properties of pedestrians, used in other crowd injury simulation studies, together into a single simulation. This design provides enhanced dynamic analysis of crowds throughout a scenario, by handling motion-altering and injury-tracking capabilities simultaneously, while also adding crowd-centric occurrences like crush.

Model data operations will be devised to test encoding crowd properties in the experimentation section. Then, the results will be used to present and evaluate the model's visual and internal runtime behaviour and performance. Finally within the discussion section, the degree of success for each study objective, and answer for each research question proposed promptly, will be addressed.

This study aims to *explore the utility of representing simulated, dynamic crowd health to guide scenario safety*, by asking these research questions: (R1) *"Is crowd health currently a suitable application of crowd simulation?"*, (R2) *"Is SPH effective for simulating crowd health?"*, and (R3) *"What are the suitable metrics involved in measuring, influencing, and ensuring the validity of health in simulated crowds?"*. Answering these will progress crowd safety advice from simulation services, enhancing crowd management studies for event organisers.

The following model study objectives aim to *ensure the stability of using crowd health dynamic analysis within a scenario.* This includes designing then compiling a validated simulator, that effectively parametrises, and observes, the elevation of panicked crowd behaviour, and elevation of crushing force on the crowd, respectively. Crowd panic will be represented by erratic pedestrian motion, while crowd crush will inform the likelihood of a pedestrian being injured.

2 Background Research

This section will introduce the individual components of research relevant during the study, including the development of agent-based simulations, representation of pedestrian crowds using agents and simulators, introduction of particle-based simulation methods into crowd simulations, and finally the formalising of simulated pedestrian behaviour pertaining to crowd health.

2.1 Crowd Simulation

Agents are autonomous, adaptive entities which can react to and influence their surroundings [1]. Later they are introduced graphically as actors in a continuous virtual space, with visual cues and motion signifying behaviour [2]. Rules of motion enact gradual changes to direction and speed, producing group-wide behaviours for co-dependent herds from microscopic-scale decisions.

Social Forces is a centralised standard mechanism specifically for representing flow of independent pedestrians within a crowd [3]. Using forces of attraction, repulsion, and tendency towards a desired speed, entities can avoid collisions with neighbours and borders while moving towards their goal.

Simulators represent a framework for executing crowd simulations, making future research more straightforward to implement. UMANS [4], or the Unified Microscopic Agent Navigation Simulator, simulates human crowd navigation using cost-based functions to represent many common techniques [5].

2.2 Smoothed Particle Hydrodynamics (SPH)

SPH is used to smooth particles' properties macroscopically to achieve a higher accuracy for dense particle simulations [6]. Originally used to approximately represent dynamically-flowing, astronomical objects like stars, SPH enabled mesh-free, decentralised and scalable computation of highly complex environments. Introduced in [7] is each component by implementational requirement, over proof of features, to produce a graphical simulation of fluid particles.

A new method represents macroscopic groups of entities of a crowd simulation using smoothed motion [8]. The simulation involves directing SPH crowds using velocity fields around a finitely-spaced map. Later, [9] produces SPH crowding for a purely mesh-free environment of purposeful entities moving towards objectives, giving pedestrians full autonomy over their movement.

In [10] SPH crowds are extensively tested using UMANS, employing existing crowd simulation techniques like contact forces and social forces to reproduce dense crowd behaviour like shockwaves. Admittedly, the previous paper has been developed further to give improved results for varied-density crowds [11]. Despite the provided documentation and presentation, this paper is avoided during the study to maintain a simple implementation of SPH crowds for the purpose of experimentation to accommodate crowd health.

2.3 Crowd Panic and Crowd Health

Panic involves altering crowd dynamics in response to negative events, causing uncoordinated motion of individuals [12]. This motion is detrimental to crowds through jamming and pressures on individuals leading to inefficiency and injury. Panic passes through crowds and causes increased desired speed, new physical interactions like pushing, uncoordinated passage through bottlenecks like doorways, and reduced initiative to seek alternative exits as people confusedly follow others. These events lead to jams and clogging, high crowd pressure, and reduced flow. Models could benefit from involving forces and pressures to simulate these effects, but must remain simple to ensure validity. Additionally, panicked crowds tend to adopt herding behaviour [13] in evacuation situations. As panic and pressure builds, granular aspects found in fluid dynamics start to parallel crowd motion, like shockwave propagation from pushing and small opening outflows. This fluid-like behaviour suggests hydrodynamic simulation with fluid friction could enhance the behaviour of panicking individuals.

Within crowd simulation is a potential objective to avoid negative effects on the crowd, ensuring safety during dangerous scenarios like emergency evacuations or congestion. Physical and even mental state can be impacted in many ways, forming components of injury modelling [14] and behaviour modelling [15]; the effort of translating human mannerisms spanning physical tolerances, social psychology and environmental awareness into a tractable agent-based framework. Employing meaningful emergent characteristics of crowds means selecting movements, senses, and responses based on, and directed towards reporting, the chosen study objectives. Common health attributes within crowd simulations include injury and nervousness of individuals.

2.4 Research Gap

There is a gap amongst this research, between representing internal health-related aspects in crowd injury simulations, like panic and crush, and simulating realistic real-time interactions between autonomous agents modelled with human behaviour. Closing this gap would involve defining a reliable, conclusive way to represent real-time effects to health in a crowd of autonomous agents, whilst ensuring the simulation remains plausible and stable.

3 Methodology

The design includes the simulator model and data used throughout the simulation, leading to a prototype implementation based on UMANS. Equations and parameters are extracted from their originating literature where possible during model formulation to ensure the closest resemblance, which is tested during validation where the simulation is compared to UMANS's accompanying video material [16] using equal parameters. Some parts of design not retrievable from literature are adapted from the UMANS implementation codebase.

Our model's external repository [17] shares the prototype's full C++ implementation complete with classes and pseudocode snippets, parameterisation, and experimental parameters alongside results given by this study. Further validation will be introduced within the implementation, where subroutines can track important metrics during experiments. The simulation is also visualised in OpenGL, for comparing to the original model and detecting emergent anomalies not noticed during model study, such as wall penetration.

The experiments will represent dynamic crowd health through various model parameters. Altered crowd behaviours will be simulated using parameter variation experiments, while simulation stability is observed using visual and data analysis. The model behaviour and execution will then be evaluated and the outcome of this study will be described.

4 Design

The following simulator is an independent re-design of UMANS used by [5] to investigate extreme-density crowds, which was developed as a platform to compare crowd simulation techniques. It features an acceleration-based model for motion, along with the SPH crowding parameterisation of later UMANS research within a simple, digestible system that is easy to understand and access.

4.1 Simulator

The simulator features a numerical, forward-Euler update loop which moves pedestrians around an environment with obstacles. Particles represent pedestrians, while particles and wall boundaries represent obstacles. Pedestrian movement is influenced by nearby particles and walls using forces.

Pedestrian particles are generated inside a given area with equal spacing, with each assigned a weight using a random number stream, describing their size and forcefulness towards other particles. Obstacles are specified using boxes the walls enclose, while uniform columns and rows of identical, static, touching particles are generated inside.

The method of movement involves accumulating each particle's acceleration, which combines the immediate influence of all chosen social, instinctive and physical behaviours. The components of this acceleration are delegated to the following techniques: *Smoothed Particle Hydrodynamics* improves crowd-based flow and enables variable pedestrian crowd acclimation; *Contact Forces* enforces personal collisions between closely-neighbouring pedestrians and avoids obstacle penetration; *Navigation Policy* moves pedestrians towards their goal. These techniques, shown by [10], provide reliable dense-crowd behaviour with improved, intuitive parametric control over crowd-wide properties like density.

Each time frame Δt_{fine}, for each pedestrian, the movement modules produce an acceleration, a^{SPH}, a^{cf} and a^{goal}, processed in equation (1). This is used to calculate the new velocity v, in equation (2), then position r, in equation (3). a_{max} and s_{max} are the upper limits for acceleration and speed respectively. Time steps are measured in seconds and are simulated in real-time, provided the software and hardware can process each step in that period.

$$a_i = minLength\left(a_{max} \cdot \frac{\hat{a}_i}{\|\hat{a}_i\|}, \hat{a}_i\right), \quad \hat{a}_i = a_i^{SPH} + a_i^{cf} + a_i^{goal} \tag{1}$$

$$v_i = minLength\left(s_{max} \cdot \frac{\hat{v}_i}{\|\hat{v}_i\|}, \hat{v}_i\right), \quad \hat{v}_i = v_i + \Delta t_{fine} \cdot a_i \tag{2}$$

$$r_i = r_i + \Delta t_{fine} \cdot v_i \tag{3}$$

4.2 Simulation Overview

The environment is a two-dimensional space where pedestrians can move around. The pedestrian and obstacle classes contain attributes for implicit modular calculations, and a box of static particles with four outward-facing walls, respectively. Pedestrian particles touching walls and other particles are repelled using contact forces, while they navigate around neighbouring particles using SPH, as the navigation policy forces them towards their goal. Fig. 1 illustrates the overall activity of the simulation process.

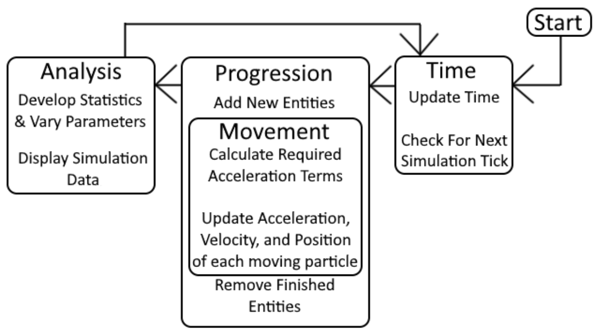


Fig. 1: Simulation loop

SPH acceleration is found by calculating the discretised Navier-Stokes partial differential equations of incompressible, viscous fluid motion. This involves finding each particle's density, then pressure and viscosity, which require a smoothing kernel, gradient, and Laplacian respectively, taken to be the "Poly6", "spiky" and "Müller" kernels used by [7]. To improve behaviour, dynamic rest density, absolute pressure and limited smoothing length are included, whilst obstacle particles only calculate density as they are static.

Contact forces acceleration is found by summating the forces involved in particle-particle and particle-wall collisions, each computed as a proportional force directly away from the collision's direction of maximum penetration depth. To improve deeply-penetrating particle-wall interactions, directly-adjacent wall contacts are prioritised, particles can only interact with one wall per obstacle, and pedestrian particles undergo a proportional repelling force even when their centre is penetrating the wall.

Navigation policy acceleration is a relaxed redirection towards the goal at a preferred speed. Pedestrians entering their goal's radius are deleted.

Visualisation draws obstacles as grey blocks, and particles as circles with a radius of their contact range, coloured using their current SPH density from blue, to green, to red, to pink as they become concentrated.

The final components don't affect the running simulation: the input component sets parameters and initialises the simulation; the debug component extracts various periodic, averaged, and dynamic-maximum-value statistical information for analysis, while varying parameters mid-run. The resulting data is later used for model validation and experimentation.

5 Experimentation

Within this section the model is validated using a base case, then exploratory experiments are defined and executed to produce results, which are evaluated to find the most promising crowd health encodings for the simulation parameters.

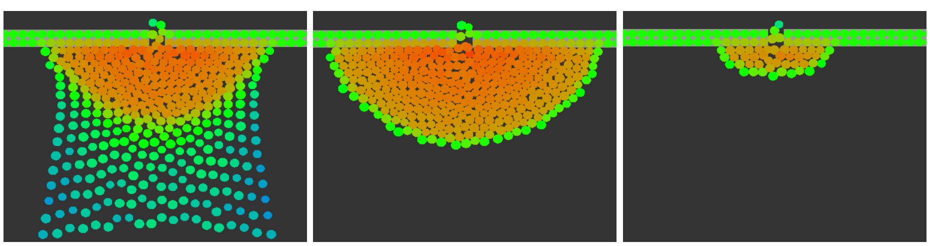
Dense crowds are generated initially to ensure the highest quality of results for this implementation. To support a preventative approach to crowd safety, negative, submissive reactions for individuals, such as passing out, are omitted from testing due to their potentially undefined effects on the simulation.

To ensure correctness and accuracy of the implementation, a base case is tweaked until the visual and tracked metrics yield stable results comparable to the UMANS video material. Visualisation exposes any stability problems through showing jittering and particle spread. To ensure consistency and higher results quality, ten uniquely, pseudo-randomly seeded simulation runs are analysed for each experiment.

5.1 Base Case (Validation Experiment)

The doorway scenario is used from van Toll's dense crowd simulation study [10], where pedestrians travel through a narrow gap whilst avoiding their neighbours and the walls, to validate the model and conduct experiments later.

Pedestrians are spread across a uniform grid within a 20m by 20m square room. An 80cm wide doorway 1m deep is across the room, with a pedestrian goal 1m into the corridor. The evacuation process starts as pedestrians collect at the entrance, slowly building in congestion then reaching an equilibrium of density and flow rate out the doorway, until the final small group rushes out. Fig. 2 shows the visualisation of these three phases of evacuation.



Collection Equilibrium Final exit

Fig. 2: Three phases of evacuation in the doorway scenario

Fig. 3 overlays the tracked statistics from ten simulation runs in a graph, for the base case scenario and parameters. The consistency of overlayed statistics suggests stability while average density, contact and speed as well as total remaining have a compelling, steady shape that could show changes easier.

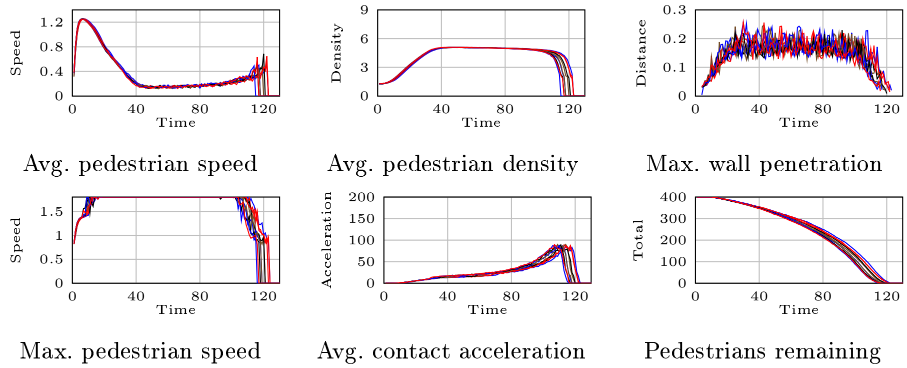


Fig. 3: Base case experiment metrics, ten replications in different colours

5.2 Exploratory Experiments

The following are parameter variations designed to represent crowd health changes, so that the results simulate emergent behaviour of crowd health disruption. Table 1 shows the most significant statistical results for the four experiments.

(a) Maximum rest density experiment. This experiment raises maximum rest density (3 to 7 between 40s and 60s) during a crowd panic, to observe how tightly average density follows rest density during variation, and compare the changing behaviour to the development of a hazardously-congested crowd. *Results:* The transition is clearly shown, from a calm crowd to a panicked crowd of high density. Rest density visually shows a steady increase, and the exit rate climbs gradually, during the variation period. Generally, the flow of pedestrians is controlled and stable, with a realistic amount of jitter before the doorway, but the final section of the exiting period appears excessively chaotic, with high amounts of jitter and a drastic rise in contact acceleration.

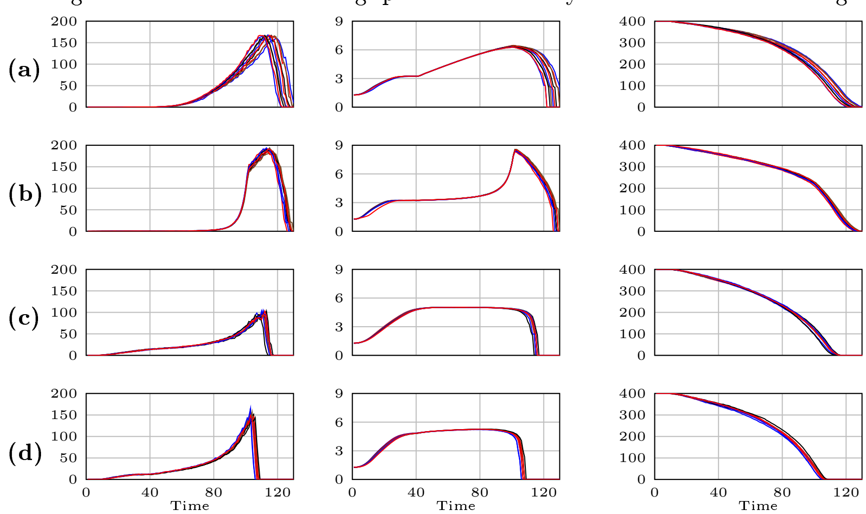
(b) SPH gas constant experiment. This experiment considers the impact of diminishing the SPH module (200 to 0 at 40s to 60s) on generating crowd disorder, without increasing the contact forces to correct for stability. This method should avoid jittery or unrealistically cramped pedestrian motions, whilst retaining simulation stability, to suit representing physically-unstable crowds in panic. *Results:* The crowd appears to show no change until a violent compression, just before the exit, causing extreme readings for density not produced in the other tests, and a surge in average contact acceleration. Following this compression, the crowd evacuates extremely quickly, disregarding realistic congestive behaviour.

(c) Preferred speed experiment. This experiment aims to represent panicked crowd behaviour by increasing desired speed (1 to 1.8 at 40s to 60s). Potentially, a higher value would cause larger reactive "pushing" forces leading to a stronger crowd and faster exit time, but raises concerns for stability, realistic density, and contact interaction quality. *Results:* Observing the collection at the doorway showed an initially slow but busy exit rate, increased in pace later, but showing similar density to the pedestrians moving through the doorway, as if the simulation was simply sped up. Jittering happens throughout but only slightly increases before the doorway, to the minimal detriment of realism.

(d) Goal force strength experiment. This experiment should represent a drive for crowd escape during a panic scenario, like preferred speed, but with a directly force-altering variable (0.5 to 2.5 at 40s to 60s). A tame panic, without an increase in average speed or jitter, is an ideal outcome for this experiment. *Results:* There is an initial calm and busy exit followed by steady density fluctuations. Frequent oscillations appear later, even with as few as thirty pedestrians to be evacuated, making the simulation seem unstable and unrealistic. The strain on pedestrians is drastically increased, but more notably the overall evacuation time is drastically reduced.

Table 1: Rows show most significant results from each experiment's findings

Avg. contact acceleration Avg. pedestrian density Pedestrians remaining



6 Discussion

Experiment **(c)** showed most potential for representing a more panicked crowd, due to the mild but controlled effect it had on contact acceleration and behaviour. It also allowed the model to retain almost all the base case's parameter values, while still allowing pedestrians to change their movement.

Experiments **(b)** and **(d)** showed that forces involved in simulation stability must not be affected mid-simulation, since this leads to violent and unpredictable behaviour which doesn't represent interactions between rigid entities including people. This is partially true for experiment **(a)**, which alters forces by indirectly reducing their effect. This causes a slight increase in jitter, but the presence of SPH allows the simulation to maintain control.

Contact acceleration was proven to portray immediate interactions of strain between pedestrians better than SPH density; density usually maintains it's threshold as average contact acceleration changes based on jittering and conflicting forces. Maximum contact acceleration might indicate crushing hazard easier since it doesn't naturally rise as open crowds dissipate. From these values, a threshold for categorising physically-dangerous crowds could be tested.

The research questions are answered below, concerning crowd health study viability and SPH's effectiveness at doing it.

(R1): Considering the complexity of emergent crowd behaviour, originating from the unpredictability of human nature and complex physical interaction, current simulators can begin to properly represent true human crowds by avoiding purely visual success criteria. This allows safety-centric data to be obtained from scenarios, especially with more access to real-world data.

(R2): SPH successfully allows nearby pedestrians to smoothly remain apart in crowds, whilst introducing an intuitive density component for affecting perception during panic. Other modules can be used to deal with direct pedestrian contacts and behaviours, to achieve more complex and simulation-stable behaviour found in real crowds.

(R3): Contact force upon an individual was found to be the most compelling measurement of crush for each pedestrian. For influencing pedestrians, affecting desired speed appeared to positively imitate a more panicked behaviour, owing to the resultant rise in erratic forces in the crowd, whilst minimising instability in the force model. To ensure validity the crowd was observed, both visually and statistically, for anomalies like jittering, unrealistic, or excessive motion whilst data outputs, including wall penetration distance, were used to tweak the base case and scenario.

The model scope and detail of experimentation had issues, preventing a more articulated conclusion. Here are some extensions which could fix such shortcomings in future work. *Model Design:* Completing van Toll's SPH model [10] by adding social forces could give parameters to represent crowd psychology and panic, whilst contact forces and SPH enforce stability in the force model, and the desired speed towards goals remains constant. A contact-acceleration-based visualisation could make more intuitive crushing crowd force observations during the simulation. Improvements from the van Toll 2021 model [11] could improve model stability for diminishing crowds. *More Scenarios:* Scenarios from recent crowd literature could be explored, some examples being a "Cluster" experiment in an open area with no obstacles, a "Passage" experiment in a congested passage with protruding checkpoints, an "Altruism" experiment for groups with differing behaviours, and an "Injuries" experiment involving altering mobility based on dynamic pedestrian health. *Experimental Parameters:* Multiple parameters could be altered within a single experiment, to represent more complex emergent crowd behaviour, and investigating parameters used in other literature could also improve the tracking of crowd health. *Crowd Safety Classification:* The ideal output of simulating a scenario should be the decision of how safe it is. Developing a framework to directly, and independently, evaluate a scenario's safety could assist ubiquitous crowd management.

7 Conclusion

A new self-contained model and prototype for a pedestrian crowd simulation has been adapted using designs from research. Using recent methods to improve stability, the prototype includes a stable base case for a moderately-panicked crowd, and methods for producing crowd panic and measuring crowd crush. Some capability and scalability of the foundational simulator was removed like the social forces module, but by altering the design and scenario scope to prioritise tight dense crowds, the results weren't harshly impacted.

A method has been presented, for concurrently analysing multiple scenario-specific crowd health types, based on current force-based crowd modelling techniques, to inform preliminary crowd management studies. However, simulated behaviours and threshold physical tolerances will need to be validated using real-world data, to provide trustworthy information about safety.

Only a prototype was developed for this study, allowing most time to be spent on model formulation and implementation maintenance, to ensure correct behaviour between the components of design. For example, reproducing the simulator involved implementing a stripped-down version of UMANS, using two-dimensional SPH kernels from [7], and setting additional constraints for walls to stop particles behaving awkwardly with the original parameters; some problems of which only presented themselves at the experimentation stage.

Contributions from the model study objectives include the designed and implemented prototype, incorporating model-level crowd-based parameters like density using recent research, along with the results and evaluation of data gained from the experiments, providing ways to represent multiple crowd-specific health properties simultaneously. Findings about representing health reliably from the four experiments were surprisingly diverse, and contribute to crowd injury simulation by separating crowd-based and simulation instability for this model. Further experimentation could streamline the production of meaningful and intuitive results, for informing safety using crowd health simulation.

During this study, we have modelled, prototyped, and validated a real-time crowd health simulation using recent crowd simulation research, then represented multiple dynamic crowd health attributes using it's foundational model's parameters. However there's much to do, like exposing this model to more scenarios and experiments, improving the model's behaviour, and applying more rigorous validation, to improve this model's ability to inform safety of crowds.

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