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Bounded confidence models generate more secondary clusters when the number of agents is growing

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Abstract We study the bounded confidence model on a growing population. We compare simulations of the agent model, its version in continuous densities and with the standard influence function or a smoother influence function. We find that the model on a growing population generates bigger secondary clusters and more systematically than when the population is fixed. Moreover, our tests with the smooth influence function suggest that these secondary clusters can be generated by a different mechanism when the population is growing than when it is fixed.

Keywords Opinion dynamics · Bounded confidence · Minor clusters · Growing population · Density model

1 Introduction

Opinion dynamics models express mathematically some hypotheses about social interactions and provide means to investigate their effect on large populations of virtual agents. For instance, bounded confidence models [1, 2] assume that when an agent's opinion is too far from the one of its interlocutor, it has no influence. This hypothesis can explain the emergence of macro-behaviours such as consensus, polarization or plurality of opinion clusters. Many papers are devoted to studying these models and their variants. For instance, several studies focus on including so-called extremists agents, whose opinion is at the border of the opinion interval [3–5], others consider agents with confidences drawn in a given interval[6], with different types of networks of interactions. Introducing noise in these models also significantly modifies their qualitative behaviour [7–9]. For a recent review on these models and related ones, see [10].

While in most of the models, the population of interacting agents is fixed, in this contribution, like in the model studied in [11], new agents are progressively added to the population. This model can be related to online communities of agents that are created, and then may grow more or less rapidly. Recent models inspired by the physics of gels address, more specifically, the dynamics of aggregation and desegregation of online groups [12]. In our model, new agents are added to the population over time while the agents are interacting, which is not the case in [11]. Our initial intention was to study this model in different network types, and particularly on scale-free networks. However, it seems indeed a necessary first step to study the model in full mixed networks in order to understand its behaviour in this simpler case and compare its results to the fixed population versions. This paper is devoted to this first step.

The simulations of the bounded confidence model on a growing population show the emergence of larger secondary clusters that are more systematically present than in the simulations on a fixed population. Indeed, the studies of the bounded confidence model with a fixed population have already identified the secondary (or minor) clusters. These clusters appear particularly clearly in the continuous version of the model [13], however they remain relatively marginal and are often ignored in the studies of the agent version of the model.

The increased importance of the secondary clusters in the model on a growing population led us to question their origin. With this aim, we also run simulations of the model with a smooth influence function. Indeed, as noticed by several scholars [3,14], the bounded confidence influence function shows a strong discontinuity when the distance of the opinions around the confidence bound, which seems difficult to justify psychologically. We propose a simple smooth influence function that erases this discontinuity. Our results suggest that, in the model on a fixed population, the secondary clusters are related to the discontinuity of the influence function and the growing population model can also generate secondary clusters through a different process.

The next section presents the model and all its versions: agent based, continuous, growing or fixed population, standard or smooth influence function. The following section is devoted to the simulation results with the standard influence function. Then we present the results with the smooth influence function. The final section proposes a discussion of these results.

2 The agent model and its continuous version

2.1 Agent model with standard influence

We consider a population of a growing number $N(t)$ of agents, with $N(0) = N_0$. An agent $i \in \{1, .., N(t)\}$ characterised by an opinion $a_i(t) \in [0, 1]$. All the agents share the same confidence bound ϵ. At each time step, we perform the classical bounded confidence interaction as in [1]. Moreover, after each interaction, with a probability $\frac{N_0 \omega}{N}$, N being the current population size and $\omega \in [0, 1]$ being a parameter, a new agent is added to the population with an opinion chosen at random uniformly in the opinion interval. More precisely, the algorithm is as follows.

1. Let T be the maximum number of iterations;
2. Initialise $N0$ agents, with opinions uniformly drawn in $[0, 1]$. Set $N = N0$;
3. For $t \in (1, ..., T)$ do:
   - Set $N_t := N$
   - Repeat N_t times:
     (a) Choose two distinct agents i and j and:

$$\text{If } |a_i(t) - a_j(t)| < \epsilon \text{ then } \begin{cases} a_i(t+1) = a_i(t) + \mu(a_j(t) - a_i(t)), \\ a_j(t+1) = a_j(t) + \mu(a_i(t) - a_j(t)), \end{cases} \tag{1}$$

     where μ is a parameter of the model, fixed to 0.5 in our simulations;
     (b) With probability $\frac{\omega N0}{N}$, add a new agent to the population with an opinion uniformly drawn in $[0, 1]$ and set $N := N + 1$.

As a result, in an iteration of N_t interactions, where each agent of the population interact once on average, the population grows of $\omega N0$ agents on average. The rationale is that we assume that the flow of incoming agents is constant over time and for each agent, the frequency of interactions for a single agent is constant whatever the population size. Therefore, the time for all the agents to make one interaction is constant whatever the size of the population and we use this time as a reference.

2.2 Continuous model

Developing a density model approximating the agent version is a classical practice (see for instance [13, 14]). The principle is to consider the evolution of a distribution of probability of presence of the agents instead of the agents themselves. The results of the continuous model are the one that would be obtained an infinite number of agents defining initially a perfectly uniform distribution and average interactions all taking place simultaneously. Therefore, comparing agent and continuous models highlights the effect of the different sources of noise in the agent model (irregularities in the initial distribution and when drawing new agents or interacting couples).

In practice, in order to run numerically the evolution of the distribution, it is necessary to cut the opinion axis into a large number M of intervals, and to consider the density of agents in each of these intervals. Therefore, the state of the system is a vector $d(t) = (d_1, ..., d_M)$ of M continuous values, approximating the continuous density. The algorithm is based on the agent model rules and computes the probabilities that the density increases or decreases in each interval. This is generally done through a master equation that expresses the sum of the inflow and outflow at each interval. The repeated action of these changes at each interval produces the evolution of the density.

Here we use an algorithm that derives the effect of the master equation by applying the average dynamics of the model and storing all the changes for a time step into a vector denoted by δ. Then all the changes of the density for an iteration are performed at once by adding δ to the current density $d(t)$. Then, we perform the addition of $\omega N0$ agents to a population. As the sum of $d_i(t)+\delta_i$, for all the intervals $i \in (1, ..., M)$, is 1, we add the vector $(\frac{\omega}{Mt}, ..., \frac{\omega}{Mt})$. Then we normalise $d(t+1)$. Finally, after the T iterations, we multiply $d(t)$ by $\frac{1+\omega t}{1+\omega T}$ for $t \in (0, ..., T)$, to get growing values of $d(t)$ like in the agent model.

The algorithm is the following:

1. Initialise $d(0) = (\frac{1}{M}, ..., \frac{1}{M})$, $t = 0$;
2. For $t \in (0, ..., T-1)$:
   (a) Initialise $\delta = (0, ..., 0)$ vector of size M;
   (b) For $i \in (1, ..., M)$
      – For $j \in (1, ..., M)$
         – if $|i - j| < \epsilon M$,
            - $k = \text{round}(i + \mu(j - i))$;
            - $\delta_i := \delta_i - d_i(t)d_j(t)$;
            - $\delta_k := \delta_k + d_i(t)d_j(t)$;
   (c) For $i \in (1, ..., M)$, $d_i(t+1) = d_i(t) + \delta_i + \frac{\omega}{M(1+\omega(t-1))}$;
   (d) $d(t+1) := \frac{d(t+1)}{\sum_{i=1}^{M} d_i(t+1)}$
3. For $t \in (0, ..., T)$, $d(t) = \frac{1+\omega t}{1+\omega T} d(t)$

2.3 Agent model with smooth influence

With the standard influence rule, the influence of agent i on agent j is strongly discontinuous when the distance of the opinions $|d_{ij}(t)| = |a_i(t) - a_j(t)|$ is around ϵ. Indeed, if $|d_{ij}(t)|$ is slightly below ϵ, the change of opinion $|a_j(t+1) - a_j(t)|$ is close to $\mu\epsilon$, which is the maximum possible change. However, if $|d_{ij}(t)|$ reaches ϵ, the change suddenly drops to 0.

This discontinuity seems difficult to justify psychologically to the eyes of some authors who proposed some changes in the influence function that eliminate this discontinuity (see e.g. [3, 14]). In this paper, we propose the following variant.

$$\text{If } |d_{ij}(t)| < \epsilon \text{ then } \begin{cases} a_i(t+1) = a_i(t) + \mu d_{ji}(t)(\epsilon - |d_{ij}(t)|), \\ a_j(t+1) = a_j(t) + \mu d_{ij}(t)(\epsilon - |d_{ij}(t)|), \end{cases} \tag{2}$$

We refer to this version as the smooth influence and to the previous version as the standard influence. The smooth influence can of course be introduced in both the agent model and in its continuous version.

3 Simulations with the standard influence function

3.1 Examples for $N_0 = 1000$ and $N_0 = 5$

Figures 1 and 2 compare model simulations, for both the agent and its continuous approximation. The density for the agent model is obtained by counting

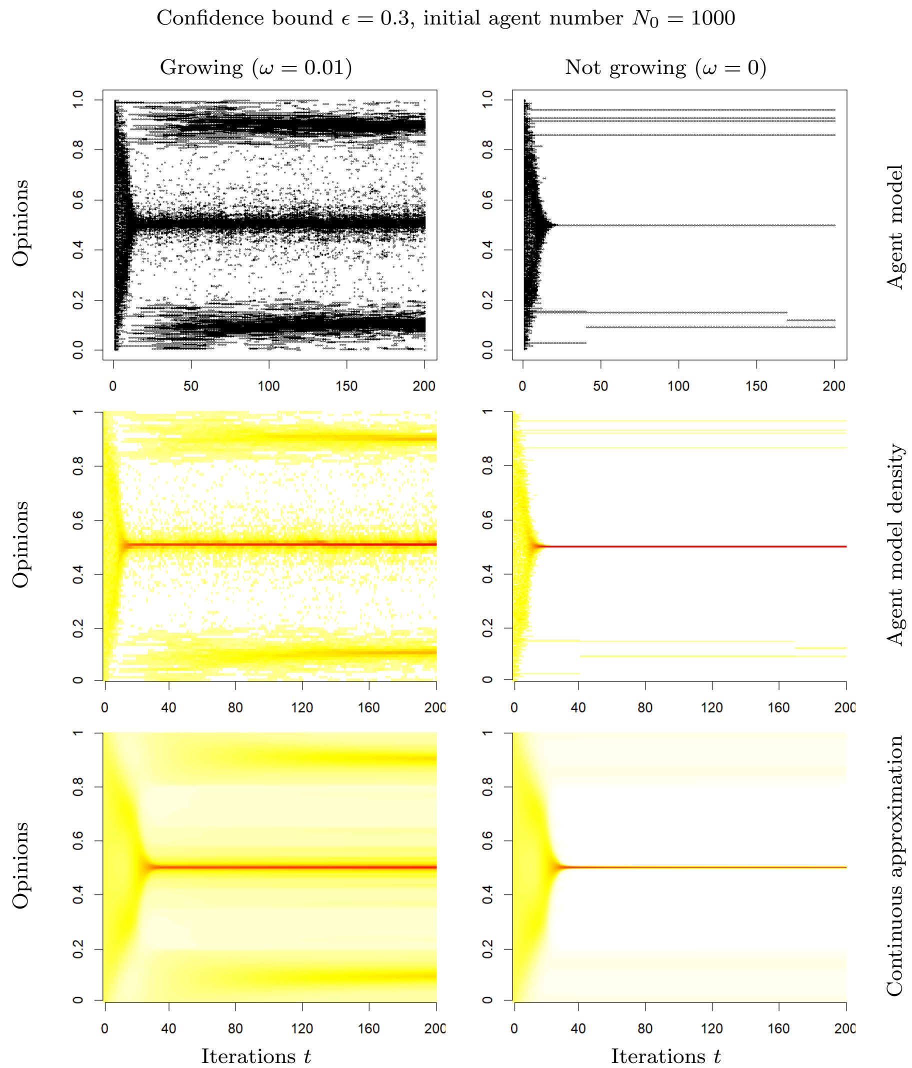

Fig. 1 Comparing growing and non-growing cases for the confidence bound $\epsilon = 0.3$. First line of panels represents directly the opinions, the second and the third represent the density of opinions. The second line is the density corresponding to the agent model and the third line the density given by the continuous approximation (see text for details).

the number of agents in 200 regular intervals of the opinion axis. This number is divided by the total number of agents at $t = 200$. Remember that the time t is incremented every N pair interactions, N being the size of the population at the beginning of time step t.

The main observations from these first examples are the following:

- In the growing population model, the secondary clusters are significantly larger than in the non-growing case.

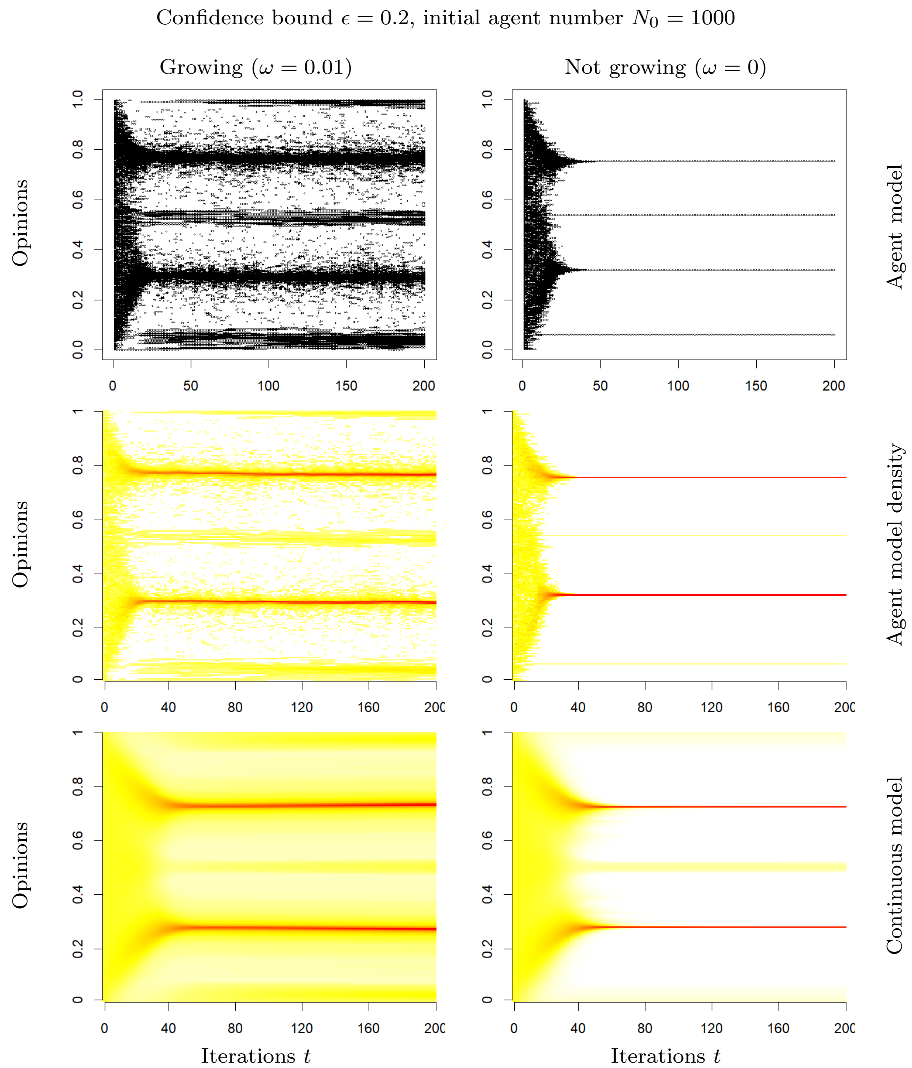

Fig. 2 Comparing growing and non-growing cases for the confidence bound $\epsilon = 0.2$. First line of panels represents directly the opinions, the second and the third represent the density of opinions. The second line is the density corresponding to the agent model and the third line the density given by the continuous approximation (see text for details).

- In the growing model, there is a density of agents remaining around the major clusters, while this density is null around the major clusters in the non-growing model.
- The results of the agent model are rather close to the results of its continuous model.

Figure 3 shows examples of runs of the model for an initial number of agents $N_0 = 5$ and adding $\omega N_0 = 5$ agents at each round (first line). On the

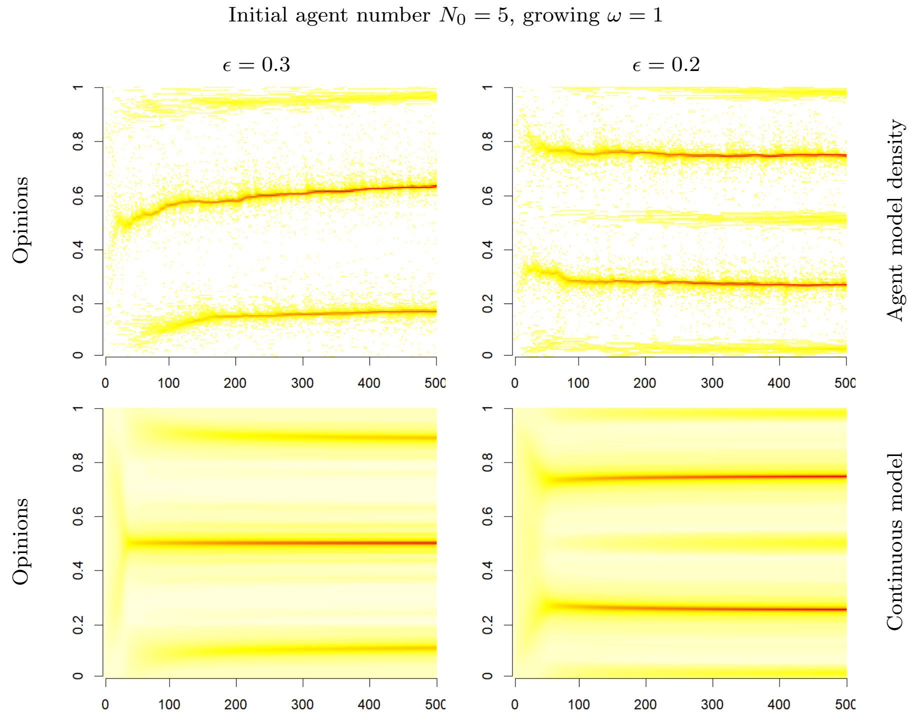

Fig. 3 Agent model and continuous approximation for initial number of agents $N0 = 5$ and adding $\omega N0 = 5$ agents at each round (see text for details).

second line, we show the continuous approximation for $\omega = 1$. Indeed, in this approximation, the initial number $N0$ cannot be taken into account, as the initial state is a perfect uniform distribution in any case.

3.2 Configurations of clusters when the confidence bound ϵ varies

Figure 4 shows the positions of the clusters for the continuous approximation model when growing ($\omega = 1$) and not growing ($\omega = 0$), as a function of $\frac{1}{2\epsilon}$. The figure distinguishes between primary, secondary and intermediate clusters. The distinction is based on the effective weight of the cluster (see the caption of figure 4). The effective weight of a cluster is defined in the appendix of this paper.

Overall the growing and non growing cases yield similar patterns of cluster positions. A few differences are however noticeable: for $\frac{1}{2\epsilon} = 2.25$, a central secondary cluster is detected in the growing case while it is not in the non-growing case. Similarly, for $\frac{1}{2\epsilon} = 3.25$, the growing case shows two secondary clusters more than the non-growing case. For $\frac{1}{2\epsilon} = 2.75$, the central cluster is secondary for the growing case, while it is intermediate for the non-growing case. Finally, the positions of the clusters are a bit different. These positions

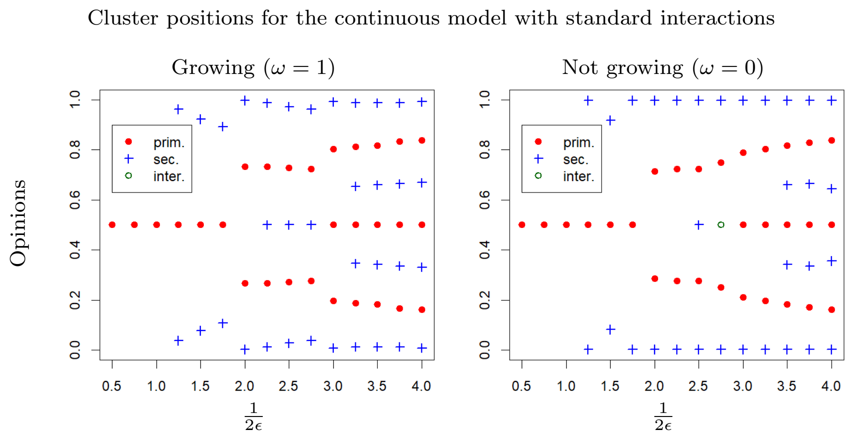

Fig. 4 Positions of clusters from the continuous approximation for both growing and non growing cases for different values $\frac{1}{2\epsilon}$. The primary (prim. in the legend) clusters a such that their power is higher than 0.6, the secondary clusters (sec. in the legend) are such that their effective weight is lower then 0.2. The intermediate clusters (inter. in the legend) are such that their effective weight is between 0.2 and 0.6. See appendix for the definition of the effective weight of a cluster.

change more sharply at the transitions in the growing than in the non-growing case.

The configurations of clusters obtained with the continuous model are references for the agent model. For a given value of the confidence bound ϵ, depending on the other parameters (N_0 and ω), the agent model can yield more or less often the same configuration as the continuous model.

The top panels of figure 5 show the average positions of the clusters for the agent based model in two cases. In one case (right panel), the initial number $N_0 = 1000$ and the average number of agents added at each iteration is $\omega N_0 = 10$ and the cluster positions are measured after $T = 200$ iterations (on average 3000 agents). In the other case (left panel), the initial number $N_0 = 5$ and the average number of agents added at each iteration is $\omega N_0 = 5$ and the cluster positions are measured after $T = 500$ iterations (on average 2500 agents). In each case, the figure shows the average of the most frequent configuration (defined by the number of primary clusters) over 100 replicas for each value of ϵ. Comparing these results with the results from the continuous model yields the following main observations:

- The average positions of clusters for $N_0 = 1000$ and $\omega = 0.01$ (left panel of figure 5) is very close to the result obtained with the continuous growing model, even though $\omega = 1$ for the continuous model. The only noticeable difference is that the agent model shows additional secondary clusters at the transitions between 2 and 3 and 3 and 4 primary clusters (for $\frac{1}{2\epsilon} \in \{2, 3\}$),
- The transitions take place at lower values of $\frac{1}{2\epsilon}$ when the initial number of agents is $N_0 = 5$ (right panel) than when $N_0 = 1000$. Indeed, for $N_0 = 5$, three transitions are observable: between 1 and 2 primary clusters, for

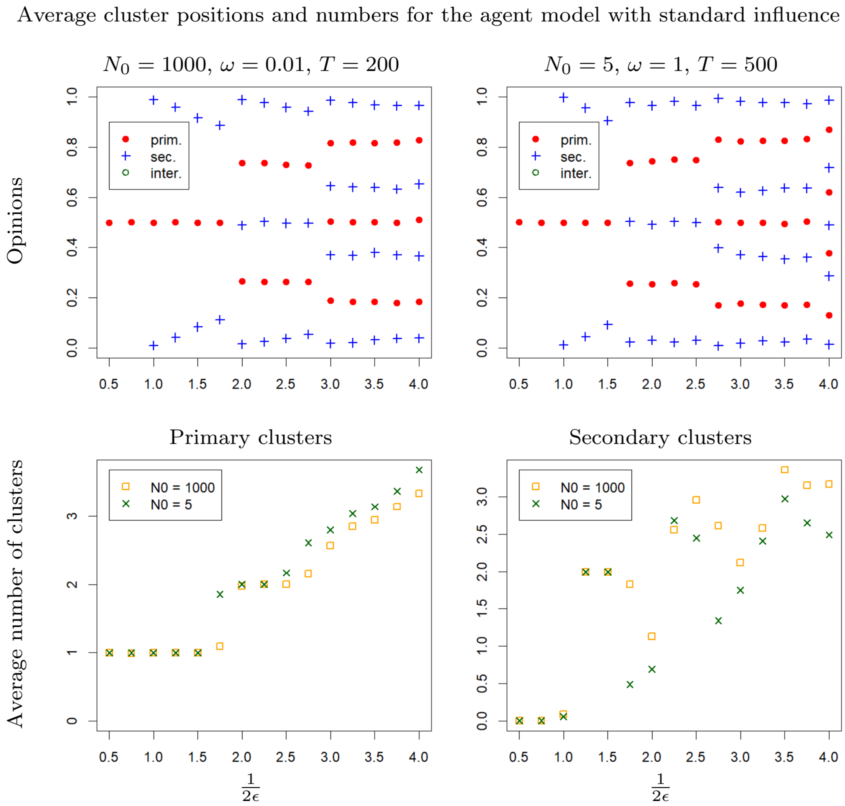

Fig. 5 Positions of clusters (top panels) and average number of clusters (bottom panels) from the agent model for initial number of agents $N_0 = 1000$, $\omega = 0.01$ and after $T = 200$ iterations (left panel) and $N_0 = 5$, $\omega = 1$ and after $T = 500$ iterations (right panel). The positions of the clusters (top panels) are the average for the most frequent configuration defined by the number of primary clusters over 100 replicas (for each value of ϵ). The average number of clusters (bottom line) is performed on all configurations. The left panel shows the average number of primary clusters, the right panel the average number of secondary clusters. The definition of the clusters with their effective weight is the same as in Figure 4.

$\frac{1}{2\epsilon} \in [1.5, 1.75]$, between 2 and 3 primary clusters for $\frac{1}{2\epsilon} \in [2.5, 2.75]$ and between 3 and 4 primary clusters for $\frac{1}{2\epsilon} \in [3.75, 4]$. For $N_0 = 1000$, only two transitions are observable: between 1 and 2 primary clusters, for $\frac{1}{2\epsilon} \in [1.75, 2]$, between 2 and 3 primary clusters for $\frac{1}{2\epsilon} \in [2.75, 3]$.

These observations suggest that, in general, a low initial number of agents N_0 leads to a higher number of primary clusters in the most frequent configuration (for a similar average number of agents ωN_0 added at each iteration).

This observation is completed by the bottom panels of figure 5 showing the average number of clusters for all the configurations. Indeed, the left panel shows that the average number of primary clusters is a bit higher for $N_0 = 5$ than for $N_0 = 1000$. However, on the contrary, the right panel shows that the number of secondary clusters is generally higher for $N_0 = 1000$. This suggests that, for $N_0 = 5$, when the the primary clusters are more numerous, they tend to be too close to each other for allowing the emergence of secondary clusters.

4 Simulations with the smooth influence function

On Figure 6, the top panels show the positions of the clusters when ϵ varies for the continuous model and the bottom panels show the simulation (growing and not growing) for $\epsilon = 0.2$. It is striking that the secondary clusters are almost absent in the non-growing case. There are some additional secondary clusters in the growing case, which are located at the extremes of the opinion interval. Moreover, the transitions between configurations of numbers of primary clusters take place at lower values of $\frac{1}{2\epsilon}$ than with the standard influence.

These results suggest that the additional secondary clusters found in the growing case with smooth influence are generated by a different process than the one generating the secondary clusters in the non-growing case, which seems mainly due to the discontinuity of the standard influence function.

Figure 7 is the equivalent of figure 5 , replacing standard influence with smooth influence. It shows the average positions of the clusters of the most frequent configuration over 100 replicas of simulations of the agent model (top panels) and the average number of primary and secondary clusters (bottom panels). The left top panel shows the results of the simulations for an initial number of agents $N_0 = 1000$ and adding on average $\omega N_0 = 10$ agents at each iteration, while the right top panel shows the results for $N_0 = 5$ and adding on average $\omega N_0 = 5$ agents at each iteration.

The average positions of the clusters of the most frequent configurations are similar in these top panels. The main difference is that the transition between 1 and 2 primary clusters takes place for $\frac{1}{2\epsilon} \in [1.5, 1.75]$ for $N_0 = 1000$, like the continuous model, and for $\frac{1}{2\epsilon} \in [1.25, 1.5]$ for $N_0 = 5$. The transitions from 2 to 3 and from 3 to 4 primary clusters take place in the same intervals $\frac{1}{2\epsilon} \in [2.25, 2.5]$ and $\frac{1}{2\epsilon} \in [3.25, 3.5]$, like in the continuous model.

The positions of the secondary clusters are not regular, probably because in general only a small part of the secondary clusters that are visible on the figures are present in each configuration participating in the average. For instance, for $N_0 = 1000$ and $\frac{1}{2\epsilon} = 3.75$, the average number of secondary clusters is 0.65. Therefore, in most of the simulations, there is only one secondary cluster. As a consequence, each position of secondary cluster is averaged on a small sample, which explains the irregularities.

The average numbers of primary clusters on all the configurations (left bottom panel) are very similar for $N_0 = 1000$ and $N_0 = 5$, the number being slightly higher for $N_0 = 5$. Note that the number grows almost linearly for $\frac{1}{2\epsilon} > 2.5$.

There are more differences in the average numbers of secondary clusters (right bottom panel). Most of the time, the average number of secondary clusters is smaller for $N_0 = 5$, which seems difficult to explain only by the small difference of the number of primary clusters. Hence this point may require

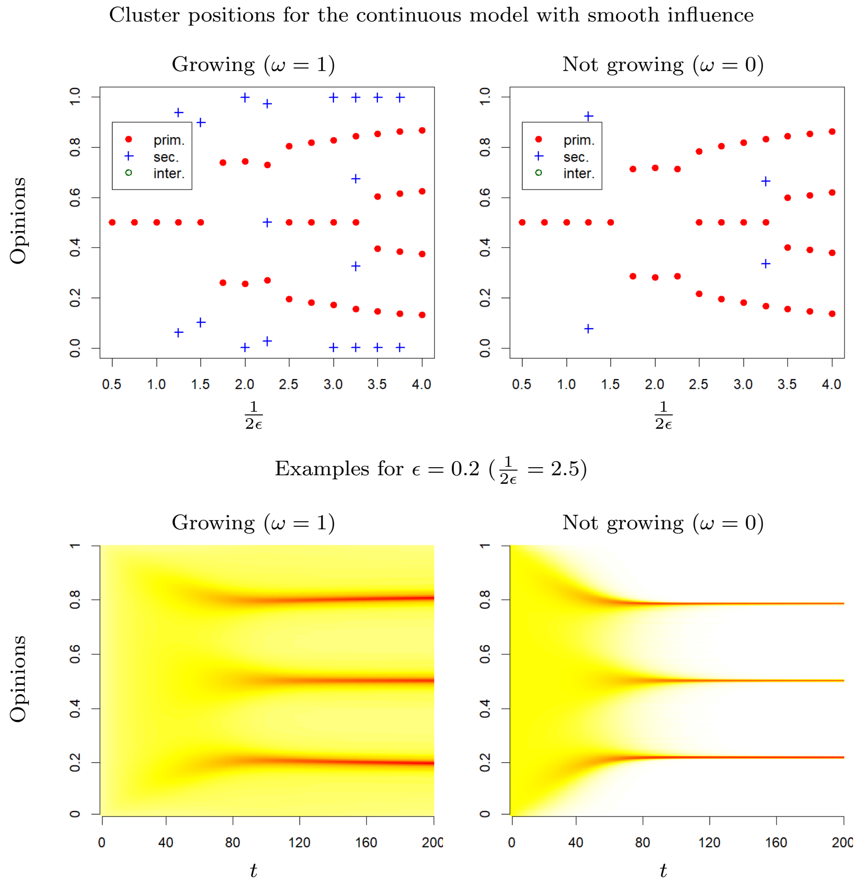

Fig. 6 Positions of clusters for the smooth influence from the continuous approximation for both growing and non growing cases for different values $\frac{1}{2\epsilon}$. The clusters are defined like previously and they are computed at the last iteration of the simulation ($T = 200$).

further investigations. Overall, the number of secondary clusters is significantly lower with the smooth influence than with the standard influence, suggesting that the secondary clusters are emerging only because of the regular arrival of new agents in locations of the opinion interval that are not under the influence of a primary cluster.

5 Discussion

This research is still ongoing. Nevertheless, it already reached some conclusions about the bounded confidence model on a growing population. In this section, we summarise the results that seem established and the remaining open questions.

Average cluster positions and numbers for the agent model with smooth influence

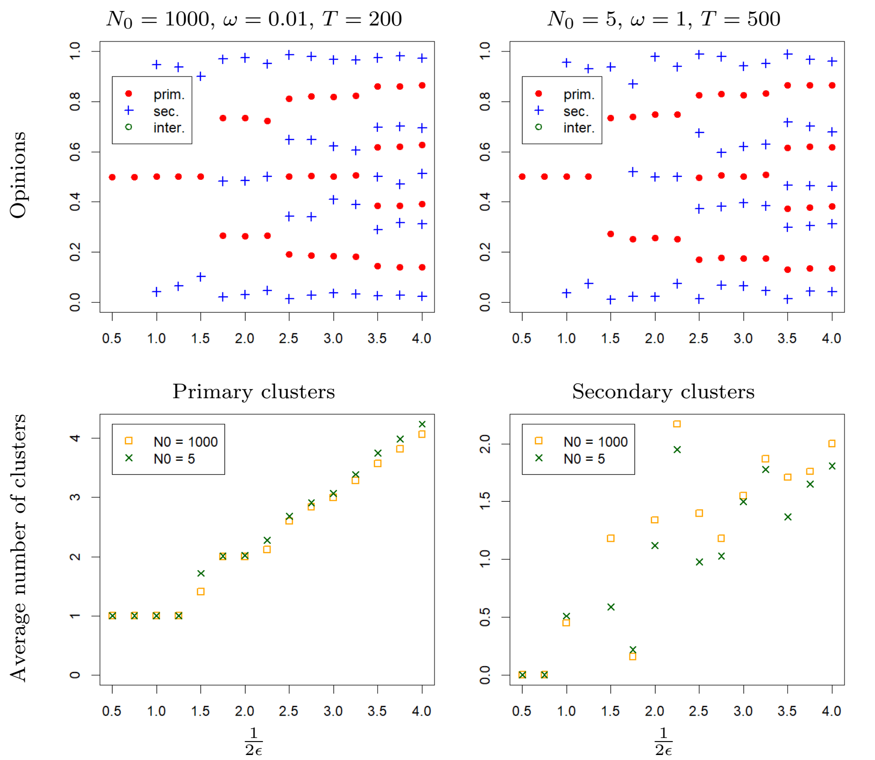

Fig. 7 Positions of clusters (top panels) and average number of clusters (bottom panels) from the agent model for initial number of agents $N_0 = 1000$, $\omega = 0.01$ and after $T = 200$ iterations (left panel) and $N_0 = 5$, $\omega = 1$ and after $T = 500$ iterations (right panel). The positions of the clusters (top panels) are the average for the most frequent configuration defined by the number of primary clusters over 100 replicas (for each value of ϵ). The average number of clusters (bottom line) is performed on all configurations. The left panel shows the average number of primary clusters, the right panel the average number of secondary clusters. The definition of the clusters with their effective weight is the same as in Figure 4.

The behaviour of the bounded confidence model on a growing population is significantly different from the noisy version of the model because the distribution of agents tends to a density with several fixed peaks (primary and secondary), while in the noisy bounded confidence, this is generally not the case [7,8]. From our simulations, it seems that the model can reach different steady states, like the model on a fixed population, depending on random events taking place in the initial phase of the simulation. However, it is difficult to check the model on very long simulations as the number of agents keeps growing.

The most striking novelty of the model on a growing population is the systematic emergence of secondary clusters, which are larger and take place more systematically than when the population is fixed. Nevertheless, when considering the continuous versions of the agent models, the maps of the primary and secondary clusters when the confidence bound varies are similar. This may suggest that the secondary clusters appear with a similar process whether the population is growing or not, the growing population simply reinforcing an existing process.

The results obtained with smooth interactions suggest otherwise. Indeed, in this case, the model on a fixed population shows almost no secondary cluster, while the secondary clusters are as still present in the simulations of the model on a growing population, albeit less systematically.

This suggests that, when the population is fixed, the secondary clusters are generated by the discontinuity of the standard influence and, when the population is growing, they come from another process. At this stage, our hypotheses about these processes are the following:

- The standard influence function shows its maximum effect at the border of the attraction basin of a shaping cluster. As a result of this strong attraction, the density of opinion is likely to be depleted inside the basin in the vicinity of its border. Therefore, the opinions located just outside the attraction basin can become isolated from the opinions within the basin are thus not being attracted. This situation does not occur with the smooth influence function, because the opinions located at the border of the attraction basin move slowly and have more chances to remain close enough to attracted opinions beyond the border while the opinions are progressively gathering into a peak.
- With both standard or smooth influence, the primary clusters are often further than 2ϵ from each other or further than ϵ from the limit of the opinion interval. This leaves some regions of the opinion interval which are not under the influence of a primary cluster. When the population is growing, the opinions that are regularly recruited in these areas, progressively feed secondary clusters.
- These secondary clusters can maintain themselves only if they are significantly smaller than their neighbouring primary clusters. Indeed, the regular arrival of new opinions in a shared zone of influence is likely to generate a lot of interactions back and froth between the clusters that bring them closer and closer until they ultimately merge. However, if one of the clusters is much smaller than the other, an opinion arriving in their common zone of influence has a very high chance to be attracted only by the bigger cluster. Therefore, the repeated back and forth interactions are very unlikely and both clusters can maintain themselves. Moreover, their difference in size keeps increasing, because the bigger cluster is more likely to attract the new agents (this process resembles the "preferential attachment" in social networks).
- Finally, we showed that varying the initial number of agents in the population leads to significantly different results. In particular, when this number is small (e.g. 5 to 10), the model shows a first phase during which the position of the primary clusters can change significantly, possibly because of

interactions with secondary clusters. These phenomena would be interesting subjects for future investigation.

6 Appendix: measuring primary and secondary clusters

The method for measuring the clusters in a distribution of opinions involves three main steps:

- Computing a smooth distribution with Gaussian kernel operator of variance about $\alpha\epsilon$ (we generally choose $\alpha = 0.1$),
- Selecting the local maxima of the smooth distribution that dominate the distribution in a vicinity of $\beta\epsilon$ 'we generally choose $\beta = 0.5$),
- Computing the effective weight of each local maximum as its weight multiplied by the effective number of clusters.

We now describe each step in more details.

6.1 Computing a smooth distribution with a Gaussian Kernel

Let $A = (a_1, ..., a_N)$ be the distribution of opinions. Let $(x_1, ..., x_p)$ be such that for $i \in (1, ..., p)$, x_i is the middle of interval $[\frac{i-1}{p}, \frac{i}{p}]$:

$$x_i = \frac{i - 0.5}{p}. \tag{3}$$

For any number x, let the Gaussian function $G(x_i, x)$ be:

$$G(x_i, x) = \exp\left(-\left(\frac{x - x_i}{\alpha\epsilon}\right)^2\right). \tag{4}$$

For each x_i, the smoothed distribution value is defined as:

$$S(x_i) = \sum_{j=1}^{N} G(x_i, a_j). \tag{5}$$

This smoothing erases strong irregularities of the histogram of opinions and provides a more accurate view of the respective weights of the clusters than when associating the clusters to the maxima of the opinion histogram.

6.2 Selecting local maximums of the smooth distribution and computing the effective weight of each of them

The couple $(x_i, S(x_i))$ defines a local maximum of the smoothed distribution if, for all $j \neq i$ such that $|x_i - x_i| < \beta\epsilon$, $S(x_j) < S(x_i)$.

Let $(x_{i_1}, ..., x_{i_m})$ be the set of values defining the local maximums and w_{i_j}, the weight of the maximum is:

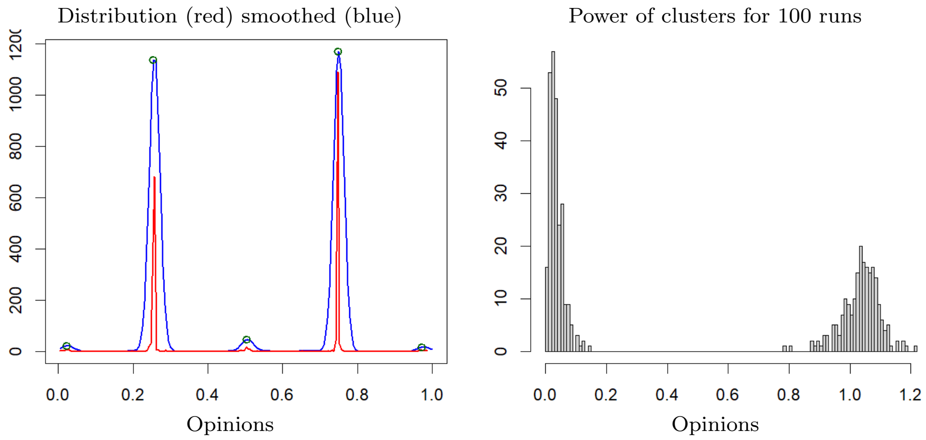

Fig. 8 On the left panel, an illustration of the method for computing the clusters. The red curve is the distribution of the opinions, the blue curve is the smoothed distribution, the dark green points are the detected local maxima. The powers of these maxima are in the order from left to right: 0.02, 1.02, 0.04, 1.05, 0.01. The right panel shows the distribution of the values of the cluster powers for 100 simulations for $\epsilon = 0.2$, $N_0 = 5$, $\omega = 1$ at $T = 100$ rounds.

$$w_{i_j} = \frac{S(x_{i_j}}{\sum_{k=1}^{m} S(x_{i_k}}. \tag{6}$$

From weights, we compute n_g the effective number of clusters [15] as follows:

$$n_e = \frac{1}{\sum_{k=1}^{m} w_{i_k}^2}. \tag{7}$$

If there are n clusters of the same weight $\frac{1}{n}$, then $n_e = n$, and if there are major clusters and minor clusters, n_e tends to be close to the number of major clusters.

The effective W_{i_j} of maximum i_j is finally defined as:

$$W_{i_j} = n_e w_{i_j}. \tag{8}$$

In general, the primary clusters define local maxima with a power around 1, while for the secondary clusters the corresponding power is lower than 0.2. In the example shown on the right pane of figure 8, there are no clusters of power comprised between 0.2 and 0.7. Therefore, the approach discriminates well between secondary (low effective weight) and primary (high effective weight) clusters.

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